

Linear Programming

1. Concept

The technique of linear programming (LP) was developed by the American mathematician G. B. Dantzig in 1946. A Russian mathematician L. V. Kantorovich also had evolved independently the technique of linear programming. The term “Linear Programming consists of two words: 1) Linear and 2) Programming. The linear programming considers only linear relationship between two or more variable. That is, it is the relationship of the type $y = a + bx$, where the relations between the variables can be represented by straight lines. Programming means planning of decision making in a systematic way.

Linear Programming refers to a technique for the formulation and solution of the problem in which some linear functions of two or more variables is to be optimized subject to a set of linear constraint, at least one of which must be expressed as inequality. It is a systematic planning or decision making which is made by the firm in relation to the production. Now-a-days, this method has been used in solving wide range of practical business problems. Linear programming has also been called **mathematical programming** and **activity analysis**.

The linear programming as applied to the theory of firm explains decision making by a firm about output and processes it will choose, given the prices of both input and products. The **Linear Programming** problem is formulated to determine the optimum solution by selecting the best alternative from the set of feasible alternatives available to the decision maker. It is a technique of selecting the best alternative out of the available set of feasible alternatives, for which the objective function and the constraint function can be expressed as linear mathematical functions.

Linear programming is used for obtaining the most optimal solution for a problem with given constraints. In linear programming, we formulate our real-life problem into a mathematical model. It involves an objective function, linear inequalities with subject to constraints.

2. Terminologies used in Linear Programming

Let us define some terminologies used in Linear Programming.

- **Decision Variables:** The decision variables are the variables that will decide the value of the variable in the objective function. They represent the ultimate solution. To solve any problem, we first need to identify the decision variables.
- **Objective Function:** It is defined as the objective of making decisions. For example, maximizing profit or minimizing cost etc.
- **Constraints:** The constraints are the restrictions or limitations on the decision variables. They usually limit the value of the decision variables.
- **Non-negativity restriction:** For all linear programs, the decision variables should always take non-negative values. This means the values for decision variables should be greater than or equal to 0.

3. Assumptions

The application of linear programming technique to any problem is based on the certain assumptions.

- i) There should be a definite objective function e.g., maximization of profit, minimization of cost.
- ii) The relationships expressed by constraints and the objective functions are linear.
- iii) The objective function is to be optimized w. r. t. the variables involved in the phenomenon.
- iv) There should be limited alternative production processes for achieving the objectives.
- v) In a linear programming problem, the firm is assumed to face a number of limitations or constraints.
- vi) It is assumed that the prices of inputs and outputs are constant.

- vii) The decision maker has certain choices, and the decision variables assume non-negative values. The non-negative assumption is true in the sense that the output in the production problem can't be negative.

Thus, while solving for the linear programming problem, these assumptions should be kept in mind such that the best alternative is chosen.

4. The process to formulate a Linear Programming problem

Let us look at the steps of defining a Linear Programming problem generically:

- a. Identify the decision variables
- b. Write the objective function
- c. Mention the constraints
- d. Explicitly state the non-negativity restriction

For a problem to be a linear programming problem, the decision variables, objective function and constraints all have to be linear functions.

5. Applications

Linear programming and Optimization are used in various industries. The manufacturing and service industry uses linear programming on a regular basis. Following are the some problems where linear programming problem has been successfully applied-

- i. **Diet Problem:** To determine the minimum requirements of nutrients subject to availability of food and their prices, this technique is used widely.
- ii. **Transportation Problem:** Optimization is also used for **optimizing Delivery Routes**. This is an extension of the popular traveling salesman problem. The service industry uses optimization for finding the best route for multiple salesmen traveling to multiple cities. With the help of clustering and greedy algorithm, the delivery routes are decided by companies like FedEx, Amazon, etc. The objective is to minimize the operation cost and time.

- iii. **Manufacturing Problem:** Manufacturing requires transforming raw materials into products that maximize company revenue. Each step of the manufacturing process must work efficiently to reach that goal. For example, raw materials must pass through various machines for set amounts of time in an assembly line. To maximize profit, a company can use a linear expression of how much raw material to use. Constraints include the time spent on each machine. Any machines creating bottlenecks must be addressed. The amount of products made may be affected, in order to maximize profit based on the raw materials and the time needed.
- iv. **Energy Industry:** Modern energy grid systems incorporate not only traditional electrical systems, but also renewables such as wind and solar photovoltaics. In order to optimize the electric load requirements, generators, transmission and distribution lines, and storage must be taken into account. At the same time, costs must remain sustainable for profits. Linear programming provides a method to optimize the electric power system design. It allows for matching the electric load in the shortest total distance between generation of the electricity and its demand over time. Linear programming can be used to optimize load-matching or to optimize cost, providing a valuable tool to the energy industry.
- v. **Workplace:** In assigning job in the production sector, the technique of linear programming problem is used highly. That is, to assign job to the workers for maximize effectiveness and optimum result subject to restriction of wages and other costs in it.