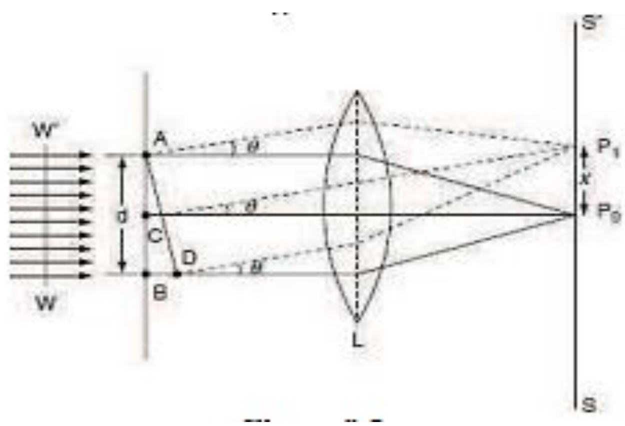
FRAUNHOFER DIFFRACTION AT CIRCULAR APERTURE

Let us consider a circular aperture of diameter d is shown as AB in figure below. A plane wave front WW' is incident normally on this aperture. Every point on the plane wave front in the aperture acts as a source of secondary wavelets. The secondary wavelets spread

out in all directions as diffracted rays in the aperture. These diffracted secondary wavelets are converged on the screen SS′ by keeping a convex lens (L) between the aperture and the screen. The screen is at the focal plane of the convex lens. Those diffracted rays traveling normal to the plane of aperture [i.e., along CP_0] are get converged at P_0.



All these waves travel some distance to reach P_0 and there is no path difference between these rays. Hence a bright spot is formed at P_0 known as Airy's disc. P_0 corresponds to the central maximum.

Next consider the secondary waves traveling at an angle θ with respect to the direction of CP_0. All these secondary waves travel in the form of a cone and hence, they form a diffracted ring on the screen. The radius of that ring is x and its center is at P_0. Now consider a point P_1 on the ring, the intensity of light at P_1 depends on the path difference between the waves at A and B to reach P_1. The path difference is $BD = AB \sin\theta = d \sin\theta$. The diffraction due to a circular aperture is similar to the diffraction due to a single slit. Hence, the intensity at P_1 depends on the path difference $d \sin\theta$. If the path difference is an integral multiple of λ then intensity at P_1 is minimum. On the other hand, if the path difference is in odd multiples

Of λ, then the intensity is maximum.

i.e., $d \sin\theta = n\lambda$, for minima ……….. **(1)**

and $d \sin\theta = (2n-1)\frac{\lambda}{2}$, for maxima ………… **(2)**

Where n = 1, 2, 3… etc. n = 0 corresponds to central maximum.

The Airy disc is surrounded by alternate bright and dark concentric rings, called the Airy's rings. The intensity of the dark ring is zero and the intensity of the bright ring decreases as we go radially from P_0 on the screen. If the collecting lens (L) is very near to the circular aperture or the screen is at a large distance from the lens, then

$$Sin\,\theta \approx \theta \approx \frac{x}{f} \qquad \text{.........................} \; (3)$$

Where, f is the focal length of the lens.

Also from the condition for first secondary minimum

$$Sin\,\theta \approx \theta \approx \frac{\lambda}{d} \qquad \text{........................} \; (4)$$

Equations (3) and (4) are equal

$$\frac{x}{f} = \frac{\lambda}{d} \; or \; x = \frac{f\lambda}{d}$$

But according to Airy, the exact value of x is

$$x = \frac{1.22 f\lambda}{d} \qquad \text{.........................} \; (5)$$

Using eqn. (5) the radius of Airy's disc can be obtained. Also from this eqn. we know that the radius of Airy's disc is inversely proportional to the diameter of the aperture.

***8**