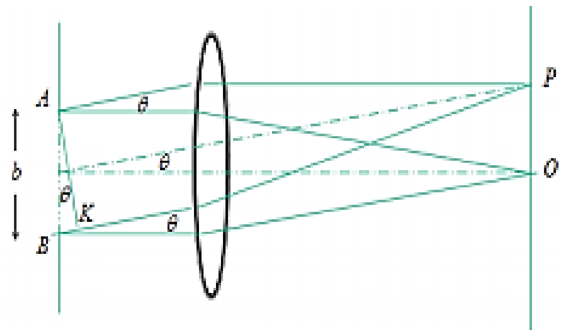
DIFFRACTION DUE TO SINGLE SLIT

Let AB is a slit of width b, the diffracted beam through the slit is tilted at an angle θ with respect to straight direction.



Path difference between two rays diffracted from two extreme points of slit

$$= BK = AB\ sin\theta = b\ sin\theta$$

Phase difference $= \frac{2\pi}{\lambda}$ x path difference $= \frac{2\pi}{\lambda}(b \sin\theta)$

Let the width AB of the slit be divide into n equal parts. The amplitude of vibration at P due to the waves from each part will be same, say a. The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} b \sin\ \theta\right) = 2\beta \text{, say}$$

Then the resultant amplitude at P is given by

$$R = \frac{a \sin(\ nd/2\ )}{\sin(\ d/2\ )} = \frac{a \sin\left(\frac{\pi b \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi b \sin\theta}{n\lambda}\right)}$$

Let us put $\left(\frac{\pi}{\lambda} b \sin\theta\right) = \alpha$

Then $R = \frac{a \sin\alpha}{\sin(\alpha/n)} = \frac{a \sin\alpha}{\alpha/n} = \frac{na \sin\alpha}{\alpha}$ .... **(1)**

When $n \to \infty$, $a \to 0$, but the product na remains finite.

Let $na = A$

The resultant intensity at P, being proportional to the square of the amplitude, is

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$ ....**(2)**

Condition for Maxima

$$R = \frac{A \sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$R = \frac{A \sin \alpha}{\alpha} = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right] \quad ....(3)$$

For $\alpha = 0$, $R = A$

This is the intensity of central maximum

$$\alpha = \left(\frac{\pi}{\lambda} b \sin \theta\right) = 0 \text{ or } \sin\theta = 0$$

Condition for Minima

$$\frac{\sin \alpha}{\alpha} = 0 \text{ or } \sin\alpha = 0, \text{ but } \alpha \neq 0$$

$\alpha = \pm m\pi$, Where m has an integral value 1, 2, 3 except zero

So $$\left(\frac{\pi}{\lambda} b \sin \theta\right) = \pm m\pi \Rightarrow b\sin\theta = \pm m\lambda \quad ....(4)$$

This equation gives the position of first, second, third etc. minima for m = 1, 2, 3 etc

Secondary Maxima

$$\frac{dI}{d\alpha} = 0$$

or $$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or $$A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha = y \text{ (say)}$$

$$y = \alpha \text{ and } y = \tan\alpha$$

The maxima will occur when

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

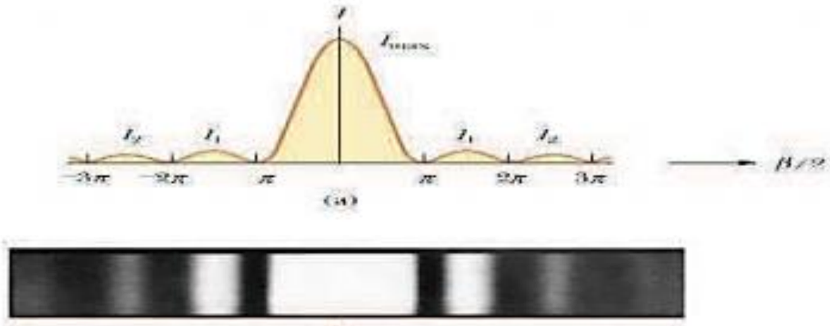
or $$\alpha = (2n+1)\frac{\pi}{2} \quad n = 1,2,3.....$$

These are points of secondary maxima

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Put $$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ etc.}$$

$$I_1 = \frac{4}{9\pi^2} I_0, \; I_2 = \frac{4}{25\pi^2} I_0, \; I_3 = \frac{4}{49\pi^2} I_0 \text{ etc}$$



I
I_max
I_2
I_1
I_1
I_2
−3π
−2π
π
π
2π
3π
β/2
(a)

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